EUCLIDEAN GEOMETRY: (±50 marks)

Grade 11 theorems:

1. ***The line drawn from the centre of a circle perpendicular to a chord bisects the chord.***
2. ***The perpendicular bisector of a chord passes through the centre of the circle.***
3. ***The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle.***
   (On the same side of the chord as the centre)
4. ***Angles subtended by a chord of the circle, on the same side of the chord, are equal.***
5. ***The opposite angles of a cyclic quadrilateral are supplementary.***
6. ***Two tangents drawn to a circle from the same point outside the circle are equal in length.***
7. ***The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.***

8. **The angle on circumference subtended by the diameter equals 90^o.**
9. **Exterior angle of a cyclic quadrilateral equals to the opposite interior angle.**
10. **A line from the centre of a circle to a tangent is perpendicular on tangent.**

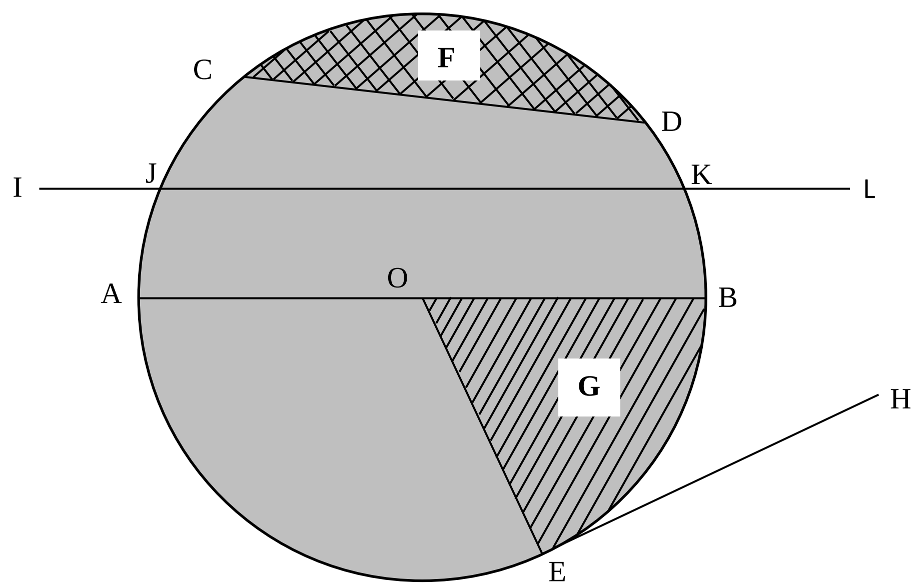
Grade 12 theorems:

1. **Proportionality theorem, (Midpoint)**

2. **Similarity theorem (Equiangular triangles)**

GRADE 11 GEOMETRY

Exercise 1
In the diagram below, O is the centre of the circle.



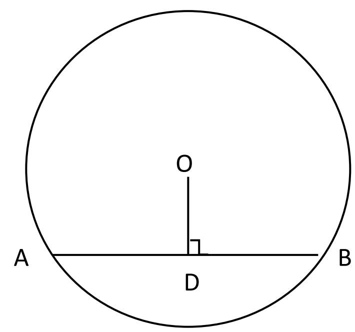
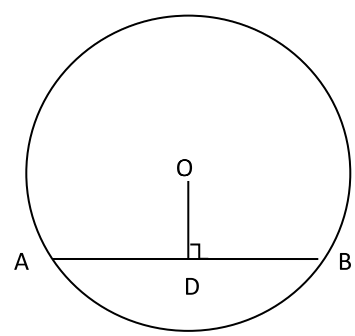
Describe the following and use the figure above to write an example of each:

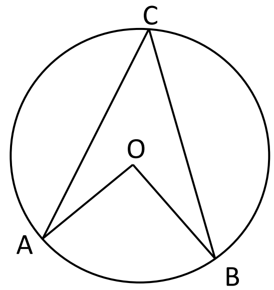
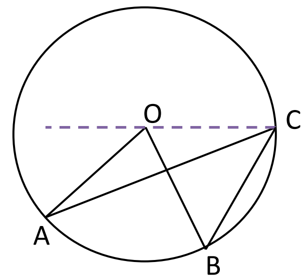
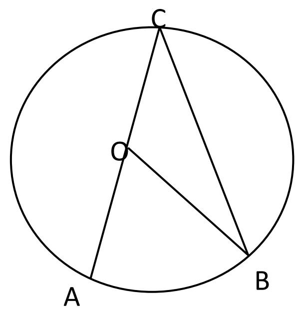
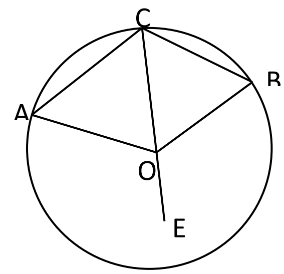
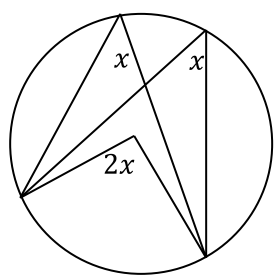
a) Diameter
b) Radius
c) Chord
d) Segment
e) Sector
f) Arc
g) Secant
h) Tangent

The following are some forms of logic applicable in proof of theorems and riders:

- If $a = b$ and $b = c$ then $a = c$
- If $a + b = c$ and $d + b = c$ then $a + b = d + b$, so $a = d$
- If $a = b + c$ and $b = d$ then $a = d + c$

According to the CAPS document there are seven theorems to be proved. The converses, where they exist, should be known to solve riders. Proofs of converses will not be examined.

Theorems	**Hints**
1. The line drawn from the centre of the circle perpendicular to the chord bisects the chord.	• Identify the information that is given and mark it on the figure.  • Write down what you aiming at, i.e. R.T.P: $AD = DB$ • Construction will lead you to congruency • Identify the given information and draw the figure. AD = DB
2. The perpendicular bisector of a chord passes through the centre of a circle.	 • How will you know that O is the centre of the circle? Which line segments should we prove equal? • Construction will help to prove $OA = OB$

3. The angle subtended by an arc at the centre of the circle is double the size of the angle subtended by the same arc at the circumference.	The following is important: • Subtended by arc / chord • Investigate angle subtended by a diameter. • Isosceles triangles and exterior angle of a triangle. • What is R.T.P.? • Construction will lead to isosceles triangles and exterior angles will assist to prove the theorem
4. Angles subtended by a chord of a circle, on the same side of the chord, are equal.	• This theorem is directly based on the previous theorem • Therefore it is important for learners to understand the previous theorem • Learners can investigate angles subtended by equal chords.

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<td>5. The opposite angles of a cyclic quadrilateral are supplementary.</td>
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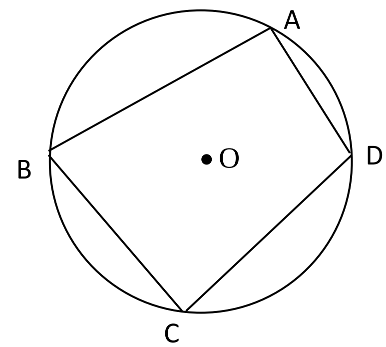
Pre-knowledge:

- Opposite
- Quadrilateral
- Cyclic quadrilateral
- Supplementary (sum $\angle s = 180^0$)

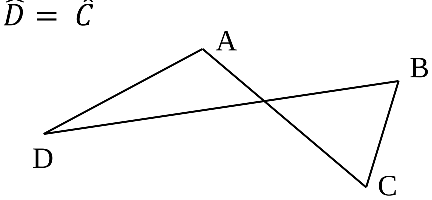


R.T.P: ABCD is a cyclic quadrilateral

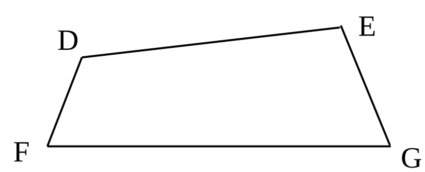
- Construction joining AO and CO can assist the learners to recognise the angle at the centre and the angle on the circle.
- Using sum of angles of a quadrilateral (360°), it can be proved that Â +Ĉ = 180°

Ways of proving that a quadrilateral is a cyclic quadrilateral:

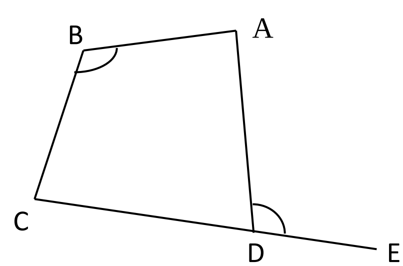
- Angles subtended by the same arc are equal, that is $\hat{A} = \hat{B}$ or $\hat{D} = \hat{C}$



- Opposite angles of a quadrilateral are supplementary



- Exterior angle of a quadrilateral isequal to the opposite interior angle.



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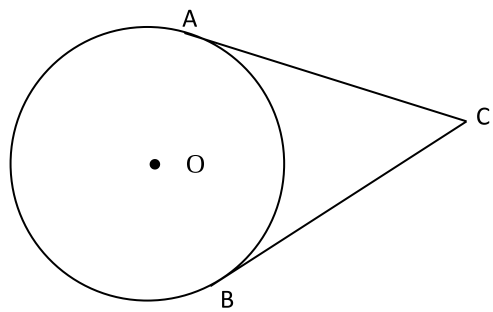
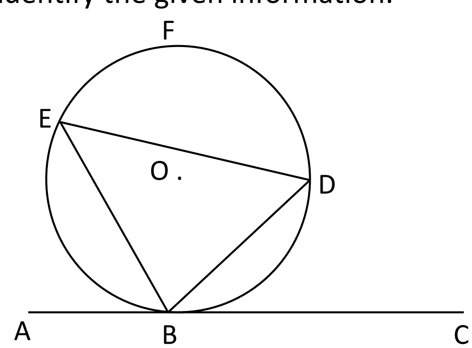
6. Two tangents drawn to a circle from the same point outside the circle are equal in length.	Revise the following: • Tangents to a circle • Radius ⊥ tangent • Congruency • Radii Identify the given and draw a figure  What is R.T.P.?By joining OA, OB and OC, it can be proved that$\Delta OAC \equiv \Delta OBC$ (RHS) $\mathrm{AC} = \mathrm{BC}$ (congruency)
7. The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	Revise theorems and axioms pertaining to: • Tangent to a circle • Identify segments and alternate segments • Angle subtended by the diameter • Sum of angles of a triangle • Diameter ⊥ tangent Draw a figure and identify the given information. R.T.P: $\mathrm{D\hat{B}C} = \mathrm{D\hat{E}B}$  Constr: Draw diameter BF and join FD and then apply the concepts.

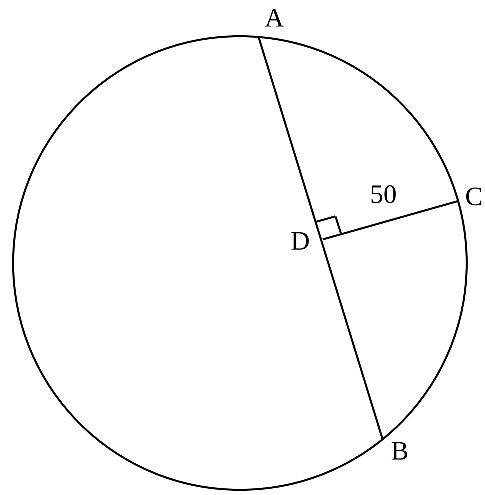
When a theorem is stated, identify:

- ❖ Information given in the statement and underline key words used
- ❖ What is to be proved.

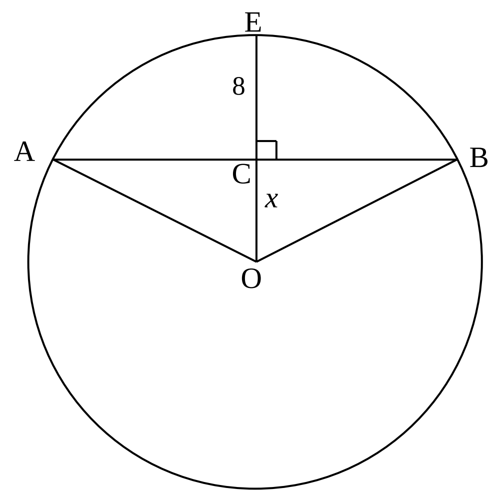
Then you need to be able to draw the sketch with the given statements and be able to what should be shown as a proof.

Exercise2

1. D is the midpoint of the chord AB and DC $\perp$ AB with C on the circle. If AB = 300mm, and DC =50mm, calculate the radius of the circle.



2. AB is the chord of the circle with centre O and is 24cm long. C is the midpoint of AB. CE $\perp$ AB cuts the circle at E. Calculate the value of x if CE = 8cm. AC = …….cm



3. AB and CD are two chords of a circle with centre O. M is on AB and N is on CD such that OM $\perp$ AB and ON $\perp$ CD. Also AB = 50mm, OM = 40mm and ON = 20mm. Determine the radius of the circle and the length of CD.

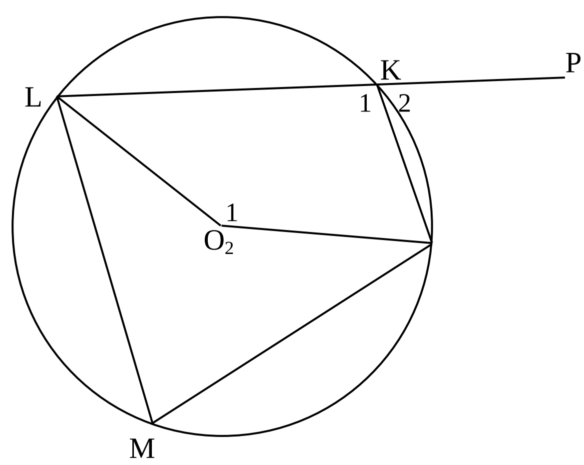
4. O is the centre of the circle below, LKP is a straight line and $\hat{O}_1 = 2x$

   4.1 Determine $\hat{O}_2$ and $\hat{M}$ in terms of x.

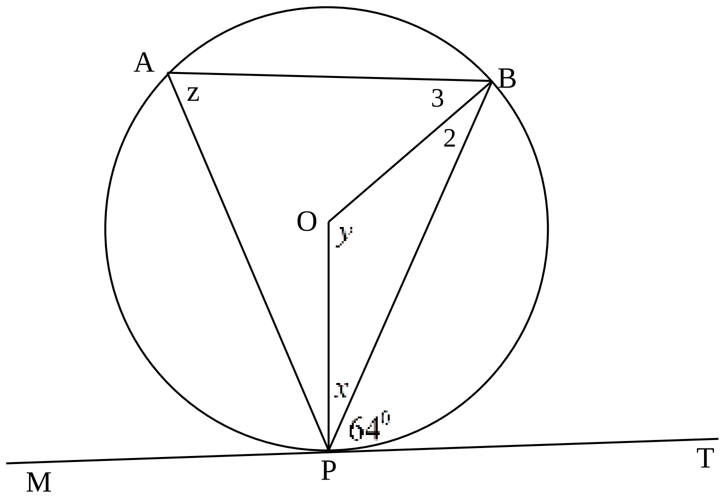
   4.2 Determine $\hat{K}_1$ and $\hat{K}_2$ in terms of x.

   4.3 Determine $\hat{K}_1 + \hat{M}$. What do you notice?

   4.4 Write down your observation regarding the measure of $\hat{K}_2$ and $\hat{M}$



5. O is the centre of the circle below. MPT is the tangent and OP $\perp$ MT.
Determine, with reasons, x, y and z

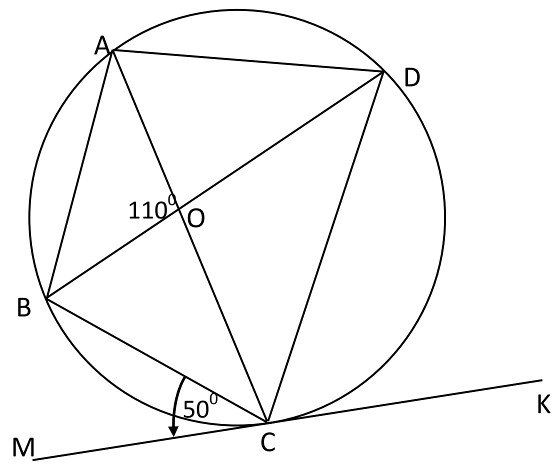


6. ABCD is a cyclic quadrilateral. MK is a tangent touching the circle at C. CA bisects B$\hat{A}$D. If AC and BD intersect at O and B$\hat{C}$M = 50^0, and B$\hat{O}$A = 110^0, calculate:

6.1 B$\hat{A}$D

6.2 A$\hat{C}$D

6.3 D$\hat{C}$K



7. PA and PB are tangents to the circle ABC at A and B respectively. PA is parallel to BC.

7.1 Prove that:

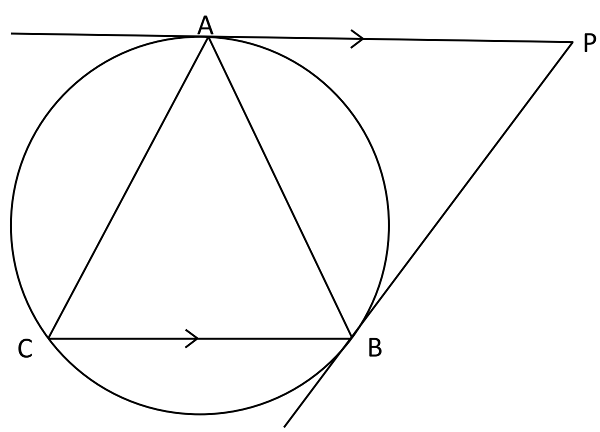
a) AB = AC

b) AB bisects P$\hat{B}$C

7.2 If A$\hat{P}$B = 40^0, determine:

a) A$\hat{C}B$

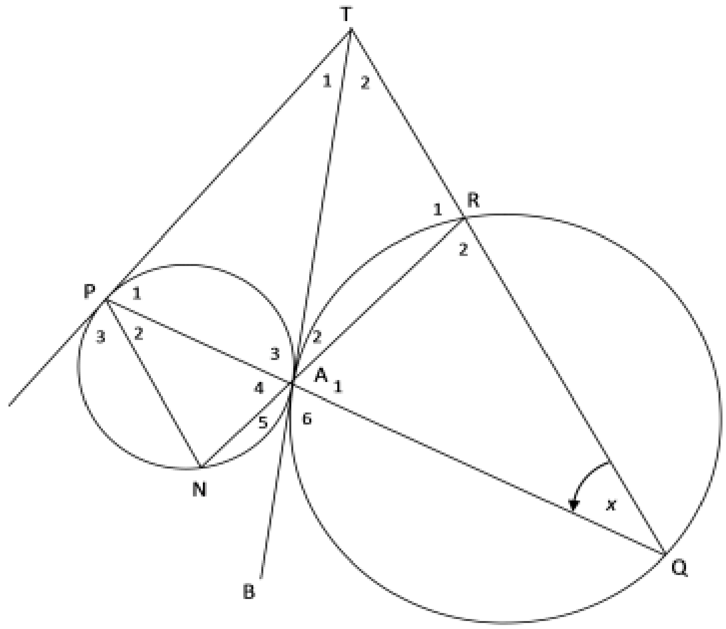
b) $B\hat{A}$C



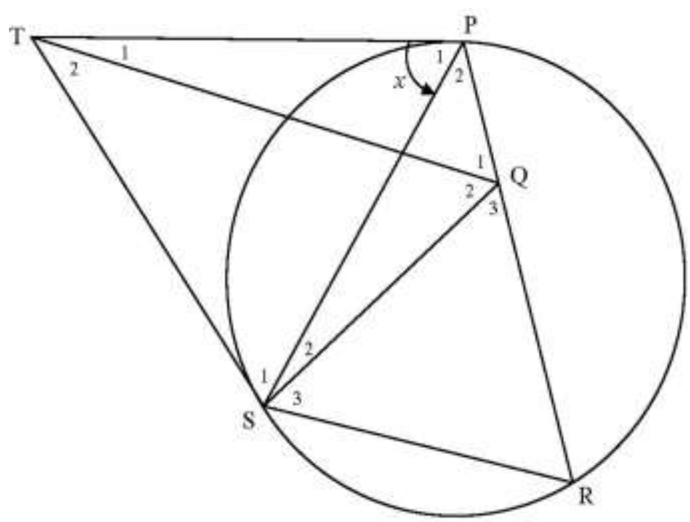
8. In the diagram below, two circles have a common tangent TAB. PT is a tangent to the smaller circle. PAQ, QRT and NAR are straight lines.Let $\hat{Q} = x$

8.1 Name, with reasons, THREE other angles equal to x.

8.2 Prove that APTR is a cyclic quadrilateral.



9. In the figure TP and TS are tangents to the given circle. R is a point on the circumference. Q is a point on PR suchthat $\hat{Q}_1 = \hat{P}_1$. SQ is drawn. Let $\hat{P}_1 = x$.



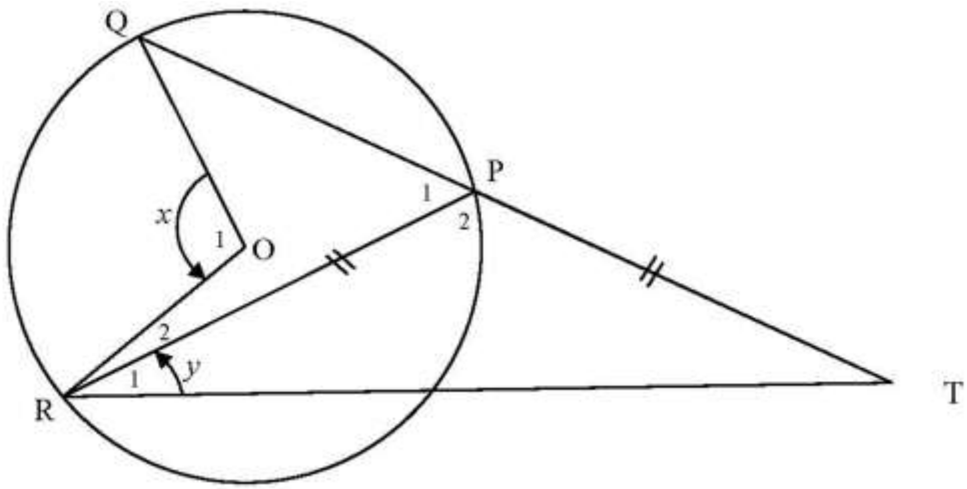
Prove that:

9.1 TQ || SR

9.2 QPTS is a cyclic quadrilateral

9.3 TQ bisects $S\hat{Q}P$

10. In the figure below, O is the centre of the circle and PT = PR.
Let $\hat{R}_1 = y$ and $\hat{O}_1 = x$.



10.1 Express x in terms of y.
10.2 If TQ = TR and $x = 120°$ calculate the measure of:
(a) y
(b) $\hat{R}_2$